Refraction of electromagnetic energy for wave packets incident on a negative-index medium is always negative

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We analyze refraction of electromagnetic wave packets on passing from an isotropic positive to an isotropic negative-refractive-index medium. We definitively show that in all cases the energy is always refracted negatively. For localized wave packets, the group refraction is also always negative.

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I. INTRODUCTION

The existence of a medium with a negative $(n<0)$ index of refraction, raised several years ago [1], has recently been demonstrated experimentally [2]. One of the most striking properties of negative index materials (NIMs) is the negative refraction for plane waves across the interface between positive-index material (PIM) and a NIM. Negative refraction means that when radiation passes through an interface between a PIM and a NIM, the refracted beam is on the same side of the normal as the incident beam (see Fig. 1), in contrast to the usual positive refraction in which they are on opposite sides of the normal.

In studies of negative refraction, it is essential to represent incident waves as localized wave packets, rather than plane waves, since all physical sources of electromagnetic waves produce radiation fields of finite spatial and temporal extent because the sources are always of finite spatial extent and because they only radiate for a finite time. Hence treatments of this problem which study waves that extend over infinite distance in all or some directions cannot be trusted to reliably predict the direction in which a wave will be refracted, and in fact treatments based on such extended waves [3] have led to a direction of refraction opposite to that which one finds for spatially localized wave packets, resulting in a great deal of controversy and confusion. Although several treatments using waves of infinite extent in some direction (e.g., a plane-wave front [4]) have obtained negative refraction, since such a model is unphysical, for the reasons given above, we cannot have confidence in conclusions obtained from it.

In this paper, we treat refraction of a localized wave packet at a PIM-NIM interface both analytically and by simulations, demonstrating that it refracts negatively. We also present both analytic and numerical studies of wave packets constructed from a small number of plane waves. Our purpose in doing this is to give a plausible explanation for why the two-plane-wave model studied by Valanju *et al.* [3] gives a misleading answer. We find that in all cases, including the model of Valanju *et al.*, the energy and momentum of the wave refract negatively. Since electromagnetic waves are detected only when they either give up energy to or exert a force on a detector, the relevant direction of propagation to consider is that of the region of space in which the energy and momentum of the wave are nonzero.

Without sources, Maxwell's equations are $\nabla\cdot\mathbf{D}=\mathbf{0}$, $\nabla\times\mathbf{H}=\partial_t\mathbf{D}$, $\nabla\times\mathbf{E}+\partial_t\mathbf{B}=0$, $\nabla\cdot\mathbf{B}=0$. For plane waves of wave vector $\mathbf{k}$ and frequency ω, only three equations are independent. Using the usual relationships between $\mathbf{D}(t)$ and $\mathbf{E}(t)$ and between $\mathbf{B}(t)$ and $\mathbf{H}(t)$ [5] one obtains for such plane waves $\mathbf{k}\times\mathbf{H}=-\omega\varepsilon(\omega)\mathbf{E}$, $\mathbf{k}\times\mathbf{E}=\omega\mu(\omega)\mathbf{H}$. Combining these equations gives us a functional relationship between ω and $\mathbf{k}$. Wave propagation is only permitted for $(\varepsilon,\mu,n>0)$ or $(\varepsilon,\mu,n<0)$ [1]. In the latter case, $(\mathbf{E},\mathbf{H},\mathbf{k})$ will form a left-handed triplet while in the former case, for an ordinary material, $(\mathbf{E},\mathbf{H},\mathbf{k})$ will form a right-handed triplet.

In Sec. II, we treat the negative refraction of wave packets and beams. The analysis of refraction of finite number of plane waves will be done in Sec. III.

II. NEGATIVE REFRACTION OF WAVE PACKETS

A wave packet localized in a compact region of space, as occurs in all experimental situations, can be constructed from a continuous distribution of wave vectors. Consider such a wave packet incident from outside the NIM, $\mathbf{E}=\hat{\mathbf{y}}E_0\int d^2k f(\mathbf{k}-\mathbf{k}_0)e^{[i(\mathbf{k}\cdot\mathbf{r}-\omega(\mathbf{k})t)]}$ with $\omega(\mathbf{k})=ck$. Here we only consider *S*-polarized waves. The *P*-polarized waves can be treated similarly, however. Throughout the paper, we choose the *z* axis from PIM to NIM normal to the interface

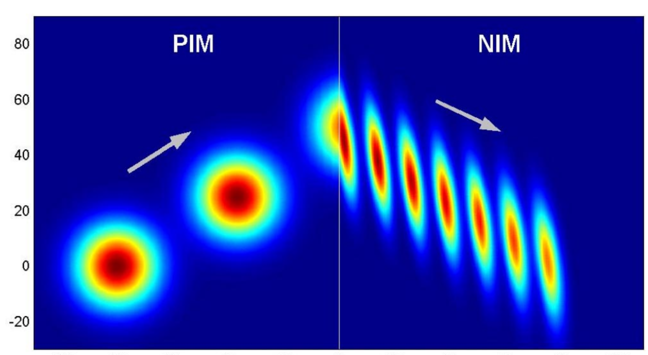

FIG. 1. Time-lapse snapshots of the electric-field intensity of a propagating Gaussian wave packet refracting negatively at a PIM-NIM interface. Arrows indicate the directions of motion. The center wave number is $k_0=\sqrt{5}$ with incident angle $\pi/6$. The spatial extent of the incident wave packet is $\Delta x=\Delta z=10$. The time step is 50 with speed of light $c=1$. The dispersion, Eq. (8), was used for NIM and $n=\mu=1$ for PIM.

and the x axis along the interface. If $f(\mathbf{k}-\mathbf{k}_0)$ drops off rapidly as $\mathbf{k}$ moves away from $\mathbf{k}_0$, $\omega(\mathbf{k})$ can be expanded in a Taylor series to first order in $\mathbf{k}-\mathbf{k}_0$ to a good approximation. This gives

$$\mathbf{E}=\hat{\mathbf{y}}E_0 e^{[i(\mathbf{k}_0\cdot\mathbf{r}-\omega(\mathbf{k}_0)t)]}g(\mathbf{r}-\mathbf{v}_g t), \tag{1}$$

with $g(\mathbf{R})=\int d^2k f(\mathbf{k}-\mathbf{k}_0)e^{i(\mathbf{k}-\mathbf{k}_0)\cdot\mathbf{R}}$ and $\mathbf{v}_g=\nabla_{\mathbf{k}}\omega(\mathbf{k})|_{\mathbf{k}=\mathbf{k}_0}$.

Inside the NIM, $\mathbf{k}$ and $\mathbf{k}_0$ in the argument of the exponent get replaced by $\mathbf{k}_r$ and $\mathbf{k}_{r0}$ which are related to $\mathbf{k}$ and $\mathbf{k}_0$ by Snell's law,

$$k_{rx}=k_x,\ \ k_{rz}=-\sqrt{(n_r\omega/c)^2-k_x^2}. \tag{2}$$

Here n_r is the refractive index for the NIM and is a function of ω. Then the wave packet once it enters the NIM is given by

$$\mathbf{E}_r=\hat{\mathbf{y}}E_0 e^{[i(\mathbf{k}_{r0}\cdot\mathbf{r}-\omega(\mathbf{k}_{r0})t)]}g_r(\mathbf{r}-\mathbf{v}_{gr}t), \tag{3}$$

where $g_r(\mathbf{R})=\int d^2k f(\mathbf{k}-\mathbf{k}_0)t_{\mathbf{k}}e^{i\mathbf{R}\cdot(\mathbf{k_r}-\mathbf{k_{r0}})}$ and $t_{\mathbf{k}}$ is the transmission amplitude for an incident plane wave of wave vector $\mathbf{k}$. It is the standard expression for this quantity for the two polarizations of the incident plane wave [6]. Here $\mathbf{k}_{r0}$ denotes $\mathbf{k}_r$ evaluated at $\mathbf{k}=\mathbf{k}_0$ and $\mathbf{v}_{gr}=\nabla_{\mathbf{k}_r}\omega(\mathbf{k}_r)$ evaluated at $\mathbf{k}_r=\mathbf{k}_{r0}$. Let us expand $\mathbf{k}_r-\mathbf{k}_{r0}$ in the exponential function in the expression for $g_r(\mathbf{R})$ in a Taylor series in $\mathbf{k}-\mathbf{k}_0$ to first order, $\mathbf{k_r}-\mathbf{k}_{r0}\approx(\mathbf{k}-\mathbf{k}_0)\cdot\nabla_{\mathbf{k}}(\mathbf{k}_r-\mathbf{k}_{r0})|_{\mathbf{k}=\mathbf{k}_0}$. Substituting this in the expression for $g_r(\mathbf{r})$, we obtain $g_r(\mathbf{R})=\int d^2k f(\mathbf{k}-\mathbf{k}_0)t_{\mathbf{k}}e^{i\mathbf{R}\cdot[(\mathbf{k}-\mathbf{k}_0)\cdot\nabla_{\mathbf{k}}(\mathbf{k_r}-\mathbf{k}_{r0})]}$. If the width of the distribution of wave vectors $f(\mathbf{k}-\mathbf{k}_0)$ is small compared to the range of $\mathbf{k}$ over which $t_{\mathbf{k}}$ varies significantly, we can to a good approximation simply evaluate this quantity at $\mathbf{k}=\mathbf{k}_0$ and put it outside the integral over $\mathbf{k}$. Then, the transmission coefficient of the wave packet is simply given by $|t_{\mathbf{k}_0}|^2$.

If we carry out the expansion of $\omega(\mathbf{k}_r)$ to second order in $\mathbf{k}-\mathbf{k}_0$, we are able to show that the wave packet spreads out, but if the length and width of the packet are much larger than the wavelength corresponding to the wave vector $\mathbf{k}_0$ at the peak in $f(\mathbf{k}-\mathbf{k}_0)$, we find that the amount that the packet spreads out in a given time interval is much smaller than the distance traveled by the packet in that time. Then clearly under such reasonable conditions, the wave packet will remain sufficiently well defined to be able to observe the refraction of the packet. The expansion of the frequency in a Taylor series is valid for a sufficiently narrow distribution, $f(\mathbf{k}-\mathbf{k}_0)$.

In order to get an explicit expression for $g(\mathbf{R})$, let the wave packet have a Gaussian form $f(\mathbf{k})=(\Delta x\Delta z/\pi)\exp[-k_x^2(\Delta x)^2-k_z^2(\Delta z)^2]$. Expanding $\mathbf{k}_r$ in a Taylor series around $\mathbf{k}_{r0}$, we get

$$g_r(\mathbf{R})=\exp[-C_x^2/4(\Delta x)^2-C_z^2/4(\Delta z)^2] \tag{4}$$

with $C_x=R_x+(cn_r/v_r-1)(k_{x0}/k_{rz0})R_z$, $C_z=(cn_r/v_r)(k_{z0}/k_{rz0})R_z$, and $v_r=c(dn_r\omega/d\omega)^{-1}$. From the above expressions, one can see that the Gaussian wave packet moves with $\mathbf{v}_{gr}$. Due to the dispersion, the wave packet is deformed in the NIM.

A NIM is dispersive and causality demands that $d(\varepsilon\omega)/d\omega>1$ and $d(\mu\omega)/d\omega>1$ for nearly transparent media [5,7]. For an isotropic NIM, since n_r is a function of ω only, $\mathbf{v}_{gr}=c(dn_r\omega/d\omega)^{-1}(c\mathbf{k}_r/n_r\omega)=-v_r\hat{\mathbf{k}}_r$ with $\hat{\mathbf{k}}_r$ the unit vector in the direction of $\mathbf{k}_r$. Since v_r is always positive for transparent media as required by causality, the group velocity will be refracted opposite to the direction of wave vector $\mathbf{k}_r$.

The magnetic field obtained from the electrical field through $\mathbf{H}=(1/\omega\mu)\mathbf{k}\times\mathbf{E}$ is

$$\mathbf{H}_r=-\frac{E_0'}{c}\int d^2k f(\mathbf{k}-\mathbf{k}_0)\frac{n_r(k)}{\mu_r(k)}(\hat{\mathbf{k}}_r\times\hat{\mathbf{y}})e^{i\mathbf{k}_r\cdot\mathbf{r}-i\omega(k)t} \tag{5}$$

with $E_0'=t_{\mathbf{k}_0}E_0$, from which we find the Poynting vector to be

$$\begin{aligned}\mathbf{S}_r&=\mathrm{Re}\ \mathbf{E}_r\times\mathrm{Re}\ \mathbf{H}_r\\&=-\frac{|E_0'|^2}{c}\int d^2k\int d^2k' f(\mathbf{k}-\mathbf{k}_0)f(\mathbf{k}'-\mathbf{k}_0)\frac{n_r(k)}{\mu_r(k)}\\&\quad\times\cos[\mathbf{k}_r\cdot\mathbf{r}-\omega(k)t]\cos[\mathbf{k}_r'\cdot\mathbf{r}-\omega(k')t]\hat{\mathbf{k}}_r,\end{aligned} \tag{6}$$

where we have used the fact that $\mathbf{k}_r\cdot\hat{\mathbf{y}}=0$. While there is no question that the Poynting vector at a point in a medium gives the local direction of energy flow, it does not give us the direction of energy flow by a wave packet or a group of plane waves as a whole since the direction of the Poynting vector varies with space. The integral of the Poynting vector over all space, $\mathbf{P}_r=\int\langle\mathbf{S}_r\rangle d\mathbf{r}$, however, gives the total momentum carried by a wave packet. This quantity divided by the volume over which the wave packet is nonzero is the average of the Poynting vector over the whole wave packet. Either way, this integral clearly represents the direction of motion of the wave packet in the medium. From the above expression for $\mathbf{S}_r$, one has

$$\mathbf{P}_r=-(|E_0'|^2/2c)\int d^2k f(\mathbf{k}-\mathbf{k}_0)^2\frac{n_r(k)}{\mu_r(k)}\hat{\mathbf{k}}_r. \tag{7}$$

Let us consider a coordinate system whose z axis is along $\mathbf{k}_0$. The function $f(\mathbf{k}-\mathbf{k}_0)^2$ will then be a function of k_x and k_z symmetrically peaked around $k_x=0$ and $k_z=k_0$. Then writing Eq. (7) as

$$\mathbf{P}_r=-(|E_0'|^2/2c)\int d^2k f(\mathbf{k}-\mathbf{k}_0)^2\frac{n_r(k)}{\mu_r(k)}\frac{k_x\hat{\mathbf{x}}+k_{rz}\hat{\mathbf{z}}}{|k_r|}$$

we can see that since the wave number k is an even function of k_x, the integrand is an odd function of k_x and hence the x component vanishes. Therefore, $\mathbf{P}_r$, which as argued above represents the propagation direction of the wave packet, is opposite in direction to $\mathbf{k}_{r0}=k_{rz0}\hat{\mathbf{z}}$, i.e., in the direction of the group velocity. Hence, the energy refracts negatively.

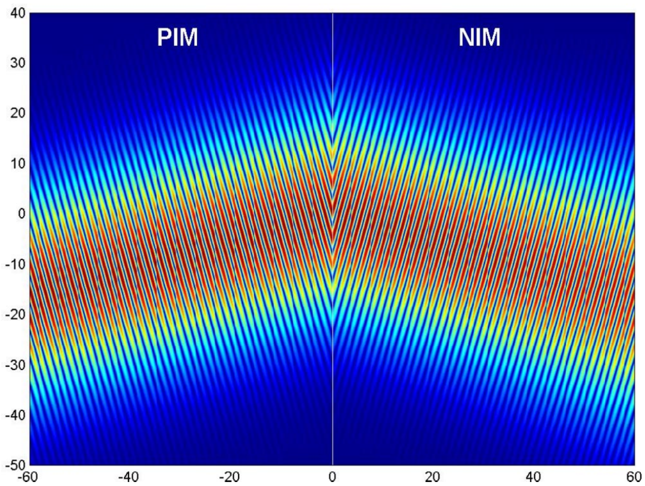

FIG. 2. Intensity of electric field Re E of a beam with $k_0=\sqrt{5}$ and the Gaussian weight $f(k_\perp)=e^{-(10k_\perp)^2}$. The incident angle of the beam is $\theta=\pi/12$.

The negative refraction of the wave packet is illustrated by numerical simulation in Fig. 1. We use the following dispersion relation

$$n_r(\omega)=-(1/\omega)\sqrt{(\omega^2-\omega_b^2)(\omega^2-\omega_p^2)/(\omega^2-\omega_0^2)} \quad (8)$$

for the NIM with $\omega_0<\omega<\omega_b$. The permeability is $\mu_r=(\omega^2-\omega_b^2)/(\omega^2-\omega_0^2)$. The numbers we used in the calculation are $\omega_0=1$, $\omega_b=3$, $\omega_p=\sqrt{10}$, and $c=1$. Figure 1 shows stroboscopic snapshots of the electric-field intensity of a propagating wave packet incident on a PIM-NIM interface [8]. The negative refraction of the wave packet is clearly evident.

For completeness, let us consider a beam given by

$$E=E_0\int dk_\perp e^{i(\mathbf{k}_0+\mathbf{k}_\perp)\cdot\mathbf{r}}f(k_\perp). \quad (9)$$

Here $\mathbf{k}_\perp$ is perpendicular to $\mathbf{k}_0$ and $f(k_\perp)$ assumes a Gaussian form. Note that this construction is different from that of Kong *et al.* [9] and Smith *et al.* [10] in that the width of the incident packet is made finite in directions perpendicular to the direction of propagation. The electric field E of the beam is shown in Fig. 2 [8]. Because the NIM is highly dispersive, the incident beam once it enters the NIM will no longer be a beam. It will be a localized wave packet instead, although it is difficult to see this in the figure. Just as for the wave packet, the beam intensity also refracts negatively.

III. NEGATIVE REFRACTION OF PACKETS CONSTRUCTED FROM A FINITE NUMBER OF PLANE WAVES

Although we have already considered the refraction of a wave packet when it enters a NIM from a PIM, we next consider the refraction of wave packets made up of a finite number of plane waves. Our reason for doing this is to provide a plausible explanation for why two plane waves example of Valanju *et al.* appears to give positive refraction. For the cases of two, and three plane waves analytical expressions are obtained for the Poynting vector, momentum, and velocity of interference pattern. First consider the case of two plane waves in the xz plane incident from PIM to NIM where the interface is at $z=0$. Let wave vectors and frequencies be $(\mathbf{k}_1,\omega_1)$ and $(\mathbf{k}_2,\omega_2)$. We set the polarization in the y direction as before. Suppose $\Delta\omega=\omega_2-\omega_1>0$. The incident wave in PIM is

$$\mathbf{E}=2E_0e^{i\mathbf{K}\cdot\mathbf{r}-i\Omega t}\cos(\Delta\mathbf{k}\cdot\mathbf{r}/2-\Delta\omega t/2)\hat{\mathbf{y}} \quad (10)$$

with $\mathbf{K}=(\mathbf{k}_1+\mathbf{k}_2)/2$, $\Omega=(\omega_1+\omega_2)/2$, $\Delta\mathbf{k}=\mathbf{k}_2-\mathbf{k}_1$, and E_0 the wave amplitude of each plane wave. The electric field of the refracted waves is

$$\mathbf{E}_r=2E_0'e^{i\mathbf{K}_r\cdot\mathbf{r}-i\Omega t}\cos(\Delta\mathbf{k}_r\cdot\mathbf{r}/2-\Delta\omega t/2)\hat{\mathbf{y}} \quad (11)$$

with $\mathbf{K}_r=(\mathbf{k}_{r1}+\mathbf{k}_{r2})/2$ and $\Delta\mathbf{k}_r=\mathbf{k}_{r2}-\mathbf{k}_{r1}$, where $E_0'=t_{\mathbf{K}}E_0$, $\mathbf{k}_{r1}$ and $\mathbf{k}_{r2}$ are related to $\mathbf{k}_1$ and $\mathbf{k}_2$, respectively, by Eq. (2).

The relatively long-wavelength cosine function in Eq. (11) moves in the NIM with a velocity [11]

$$\mathbf{v}_r=(\Delta\omega/|\Delta\mathbf{k}_r|^2)(\Delta k_x\hat{\mathbf{x}}+\Delta k_{rz}\hat{\mathbf{z}}), \quad (12)$$

assuming that $|\Delta\mathbf{k}|\ll|\mathbf{K}|$. From the above expression, it is evident that $v_{rx}>0$ if $\Delta k_x>0$. Since $\omega_1<\omega_2$, we have $0<n(\omega_1)\leqslant n(\omega_2)$ and $n_r(\omega_1)<n_r(\omega_2)<0$ by the requirement of causality which requires $d(n_r\omega)/d\omega>0$. One has $k_{r1z}^2-k_{r2z}^2=k_{r1}^2-k_{r2}^2+k_{2x}^2-k_{1x}^2>0$. Since $k_{rz}=-|k_{rz}|$, $v_{rz}>0$, the group refraction appears to be positive [3]. This is due to the simple fact that $v_{rx}>0$ if $v_x>0$. Proper dispersion must give $v_{rz}>0$ since the energy should propagate away from the interface. But we shall see that the above picture is not true for the energy flow.

Let us determine the time-average Poynting vector $\langle\mathbf{S}_r\rangle$. Using the magnetic field corresponding to $\mathbf{E}_r$ of Eq. (11), $\mathbf{H}_r=E_0'\Sigma_{j=1}^2(k_{rjz}\hat{\mathbf{x}}-k_{jx}\hat{\mathbf{z}})e^{i\mathbf{k}_{rj}\cdot\mathbf{r}-i\omega_jt}/\omega_j$, $\langle\mathbf{S}_r\rangle$ is defined to be the time average of $\mathbf{S_r}$ over the period corresponding to the average frequency of the two plane waves. It is given by

$$\langle\mathbf{S}_r\rangle=-\frac{1}{2}(1+\cos\Delta\phi_r)|E_0'|^2\sum_{j=1}^{2}\left(\hat{\mathbf{x}}\frac{k_{jx}}{\omega_j}+\hat{\mathbf{z}}\frac{k_{rjz}}{\omega_j}\right), \quad (13)$$

where $\Delta\phi_r=\Delta\mathbf{k}_r\cdot\mathbf{r}-\Delta\omega t$. Since $k_{rz}<0$, one has $S_x<0$ and $S_z>0$. Thus, contrary to the refraction of the cosine function in Eq. (11), the Poynting vector is directed in the negative refraction direction, i.e., refracts negatively.

We shall now demonstrate that by including more plane waves in our group, one can get negative refraction of the group in addition to the energy. Actually, just one additional plane wave can achieve that. Thus, let us include three plane waves, whose wave vectors form a triangle, rather than being parallel. Let the magnitudes of the wave vectors be k, $k+\delta k_1$, $k+\delta k_2$, and their angles with the normal to the interface, be θ, $\theta+\delta\theta_1$, $\theta+\delta\theta_2$. Inside the PIM or the NIM, we have

$$\mathbf{E}=\hat{\mathbf{y}}e^{i(k_x x+k_z z-\omega t)}\{1+\exp[i(u-ct)\delta k_1+iv k\delta\theta_1]+\exp[i(u-ct)\delta k_2+iv k\delta\theta_2]\}, \tag{14}$$

with $u=x\sin\theta+z\cos\theta$ and $v=x\cos\theta-z\sin\theta$ for the PIM and $u=x\sin\theta+az$, $v=x\cos\theta+bz$, and k_z replaced by k_{rz} for the NIM. Then the lines whose equations are $u=$constant and $v=$constant are perpendicular for the PIM. Here use has been made of the following expansion:

$$k_{rz}(k+\delta k,\theta+\delta\theta)\approx k_{rz}+a\,\delta k+bk\,\delta\theta \tag{15}$$

with

$$a=k(\sin^2\theta+c|n_r|/v_r)/|k_{rz}|,$$

$$b=k\sin 2\theta/2|k_{rz}|.$$

Here $v_r=c(dn_r\omega/d\omega)^{-1}$. The dependence of k_{rz} on k and θ is obtained from Eq. (2). The condition for maximum intensity for the quantity in brackets, the long-wavelength envelope of the packet, is determined by the equations

$$(u-ct)\delta k_1+vk\,\delta\theta_1=2m_1\pi,$$

$$(u-ct)\delta k_2+vk\,\delta\theta_2=2m_2\pi,$$

whose solution in the PIM is

$$x=(c_1\sin\theta+c_2\cos\theta)+\sin\theta\,ct,$$

$$z=(c_1\cos\theta-c_2\sin\theta)+\cos\theta\,ct$$

with

$$c_1=2\pi(m_2\delta\theta_1-m_1\delta\theta_2)/(\delta\theta_1\delta k_2-\delta\theta_2\delta k_1),$$

$$c_2=2\pi(m_1\delta k_2-m_2\delta k_1)/[k(\delta\theta_1\delta k_2-\delta\theta_2\delta k_1)],$$

which are clearly only defined for $\delta k_1/\delta k_2\neq\delta\theta_1/\delta\theta_2$.

Inside the NIM, the solution for the location of the intensity maxima is

$$x=(c_2a-c_1b-bct)/(a\cos\theta-b\sin\theta),$$

$$z=(c_1\cos\theta-c_2\sin\theta+\cos\theta\,ct)/(a\cos\theta-b\sin\theta).$$

From the expressions for a and b under Eq. (15), one has $a,b>0$ and $a\cos\theta-b\sin\theta>0$. Then from the above expressions of $x(t)$ and $z(t)$, one has $dx/dt<0$ and $dz/dt>0$. Thus the *refraction will be negative*. Let the angles of the line $u=$constant and $v=$constant in the NIM with the z axis be α and β, respectively. Then one has $\tan\alpha=-a/\sin\theta$, and $\tan\beta=-b/\cos\theta=k_x/k_{rz}$. So one always has $\pi/2<\alpha<\beta<\pi$ inside the NIM. From the above expressions, one can see that the maxima move in the β direction, that is, antiparallel to $\mathbf{k}_r$. The velocity of interference pattern in the NIM defined as the velocity of maximum is given by

$$\mathbf{v}_r=-v_r\hat{\mathbf{k}}_r. \tag{16}$$

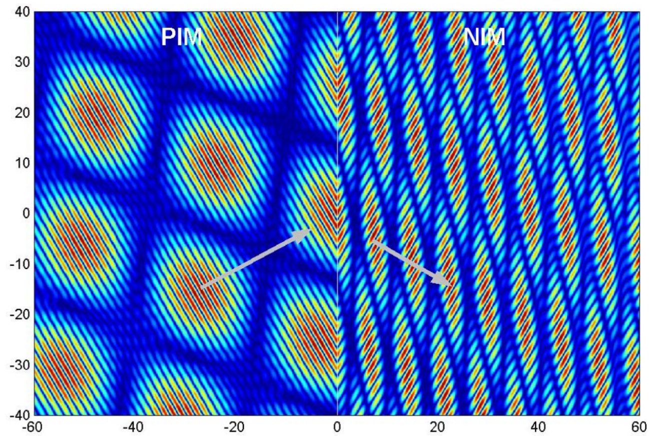

FIG. 3. Electric field Re E of negative refraction of four plane waves with wave-vector magnitudes $k-\delta k$, k, $k+\delta k$, k, and incident angles θ, $\theta-\delta\theta$, θ, $\theta+\delta\theta$, respectively. Arrows indicate the directions of motion. The center wave number is $k=\sqrt{5}$ with incident angle $\theta=\pi/6$, $\delta k=0.2$, and $\delta\theta=\pi/45$. Up to the first-order approximation, the electric field, Poynting vector, and the momentum of this group of plane waves are $E_r=2e^{i\phi_r}(\cos\varpi+\cos\delta\phi_r)$, $\langle\mathbf{S}_r\rangle=-2(\cos\varpi+\cos\delta\phi_r)^2\mathbf{k}_r/\omega$, $\mathbf{P}_r^{\mathrm{cell}}=-2A\mathbf{k}_r/\omega$, respectively.

This velocity is independent of how the incident wave packet is constructed. The refraction of a group constructed from four plane waves is shown in Fig. 3 [8]. The arguments presented above demonstrate that for any *group consisting of three or more plane waves whose wave vectors are not collinear, the group refraction is negative*.

While the simulations in Fig. 3 clearly show that the intensity maxima refract negatively, the normal to the planes in which these intensity maxima lie are directed in a positive refraction direction. Thus, if one was to imagine smoothing out all intensity variation in the planes, the planes would appear to refract in a positive direction. We believe that this is a remnant of the positive refraction of the planes of intensity maxima [the cosine function in Eq. (11)] found for the interference pattern for the two plane waves example of Ref. [3]. When there are only two plane waves, this is the only group motion that we see in the NIM since for a group consisting of two plane waves, there are no intensity variations in these planes.

Let us also look at the energy flow which is represented by the Poynting vector. For three wave vectors with wave-vector magnitudes $k-\delta k$, k, $k-\delta k$, and the angles with the normal $\theta-\delta\theta$, θ, $\theta+\delta\theta$, respectively, the magnetic field can also be calculated from Eq. (14) using Maxwell's equations and the resulting Poynting vector up to the first order in both δk and $\delta\theta$ is given by

$$\langle\mathbf{S}_r\rangle=-\tfrac{1}{2}(1+4\cos^2\varpi+4\cos\varpi\cos\delta\phi_r)\mathbf{k}_r/\omega-k\,\delta\theta\sin\varpi\sin\delta\phi_r(\cos\theta\hat{\mathbf{x}}+b\hat{\mathbf{z}})/\omega+2\,\delta k\cos^2\varpi(a-k_{rz}/k)\hat{\mathbf{z}}/\omega, \tag{17}$$

where

$$\varpi=(bz+\cos\theta x)k\delta\theta,$$

$$\delta\phi_r=(az+\sin\theta x)\delta k-\delta\omega t.$$

The time average is performed over the period corresponding to the average frequency of the three plane waves. Here, $\langle\mathbf{S}_r\rangle$ is not localized; rather it forms a lattice. A unit cell is defined as the region in which ϖ changes by π and $\delta\phi_r$ changes by 2π, as is obvious from the expression for $\mathbf{E}_r$ or $\langle\mathbf{S}_r\rangle$. The area for each unit cell in NIM is

$$A=2\pi^2/(a\ \cos\theta-b\ \sin\theta)k\delta k\delta\theta.$$

Instead of integrating over all space which will diverge, one can calculate the electromagnetic momentum for each cell. Ignoring higher-order terms in δk and $\delta\theta$, we get

$$\mathbf{P}_r^{\mathrm{cell}}=-3A(1+2\delta k/3k)\mathbf{K}_r/2\omega \tag{18}$$

with $\mathbf{K}_r=\mathbf{k}_r-2(\hat{\mathbf{x}}\ \sin\theta+\hat{\mathbf{z}}a)\delta k/3$, the average of the three wave vectors which make up the group.

A packet constructed from a finite number of plane waves will always give a collection of propagating wave pulses with the area of the unit cell inversely proportional to δk and $\delta\theta$. For the above localized waves made of finite number of plane waves, the group velocity $\mathbf{v}_{gr}$ is parallel to $\mathbf{P}_r$ and antiparallel to the average wave vector $\mathbf{K}_r$.

IV. CONCLUSION

In this paper, we have shown that for any localized wave packet, the refraction at an interface between a PIM and a NIM is always negative. As pointed out earlier, it is essential for a correct treatment of this problem to use wave packets which are localized in all directions since the electromagnetic field from any physical source is a localized wave packet.

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[8] See EPAPS Document No. E-PLEEE8-69-107401 for three avi format animations corresponding to Figs. 1, 2, and 3 of this paper. The first shows negative refraction of a Gaussian wave packet, the second simulates negative refraction of a Gaussian beam, and the third shows negative refraction of four plane waves. A direct link to this document may be found in the online article's HTML reference section. This document may also be reached via the EPAPS homepage (http://www.aip.org/pubservs/epaps.html/) or from ftp.aip.org in the directory /epaps/. See the EPAPS homepage for more information.

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